Single Pure – Modulus Function

Solve the following equations.

1. $|2x| = 7$. $x = \pm\frac{7}{2}$
2. $|x| + 10 = 0$. No solutions
3. $|x + 6| = 7$. $x = 1$ or $x = -13$
4. $2|x - 5| + 1 = 3$. $x = 4$ or $x = 6$
5. $|x - a| = 1$. $x = a - 1$ or $x = a + 1$
6. $|x + 2| = |x + 3|$. $x = -\frac{5}{2}$
7. $|3x - 2| = |7x - 1|$. $x = -\frac{1}{4}$ or $x = \frac{3}{10}$
8. $|3x - 2| = |7x - 1| + 1$. $x = 0$ or $x = \frac{1}{5}$
9. $x + 2 = |x| + |2x - 1|$. $x = -\frac{1}{4}$ or $x = \frac{3}{2}$
10. $2 + |x| = x + |x - 3|$. $x = -1$ or $x = 1$ or $x = 5$
11. $|x - 2| + 2x = |4x + 1|$. $x = \frac{1}{3}$ or $x = -\frac{3}{5}$
12. $-\frac{1}{2}x = |x| - |x + 1|$. $x = -2$ or $x = -\frac{2}{3}$ or $x = 2$
13. $|x| + 2 = 2 - |2x - 1|$. No solutions
14. $|2x - 3| + x = |x + 4| - 1$. $x = 0$ or $x = 3$
15. $|3 - 2x| + x = 6|x + 2|$. $x =$ or $x = -\frac{5}{2}$
16. $|x - 2| + |x + 2| = |\frac{x}{2}| + \frac{7}{2}$. $x = -\frac{7}{3}$ or $x = -1$ or $x = 1$ or $x = \frac{7}{3}$
17. $x = |-x| + |x| - |2x - 3|$. $x = -3$ or $x = 1$ or $x = 3$
18. $7 - |x + 3| = 2|x - 2| + |x - 1|$. $x = \frac{1}{2}$ or $x = \frac{9}{4}$
19. $|x + 1| - |x| + 3|x - 1| - 2|x - 2| = x + 2$ [USSR Olympiad]. $x = -2$ or $x \geqslant 2$

Sketch the following graphs.

1. $y = |x - 3|$.
2. $y = |2x + 1|$.
3. $y = |x - 4| - 4$.
4. $y = |x| - |x - 2|$.
5. $y = |x| + |x - 2|$.
6. $y = |x| - |x + 3| + |3x - 1|$.
7. $y = |x^2 - 4|$.
8. $y = |x^2 - 4| - |x^2 - 1|$.
9. $y = |x^2 - 4| + |x^2 - 1|$.
10. $y = |x^3 - x| - |x^3|$.

Solve the following inequalities.

1. $|2x + 3| \leqslant |x - 4|$. $-7 \leqslant x \leqslant \frac{1}{3}$
2. $2 - |x| < |x - 2| + |2x| - |x + 4|$. $x < \frac{4}{5}$ or $x > \frac{8}{3}$
3. $|x| - |2x + 3| - |x - 5| \leqslant -\frac{x}{2} - 7$. $x \leqslant -2$ or $-\frac{2}{3} \leqslant x \leqslant 2$ or $x \geqslant 6$
4. $|3x| - |x - 3| - |x + 4| > 2x - 6$. $x < -\frac{1}{5}$ or $1 < x < 5$
5. $|x^2 + x| - |x^2 - x - 2| > x$. $-2 < x < -\frac{2}{3}$ or $x > \frac{1+\sqrt{17}}{4}$
6. $x + 4 \geqslant |x^2 - 1| + |x^2 - 4|$. $x = -1$ or $\leqslant x \leqslant$

And finally…

1. Find the value(s) of c such that $|x| - |x - 2| = x + c$ has three distinct solutions.
2. Solve the simultaneous equations $|x| + |y| = 32$ and $|x| - |y| = 16$.